Coupled Pendulums

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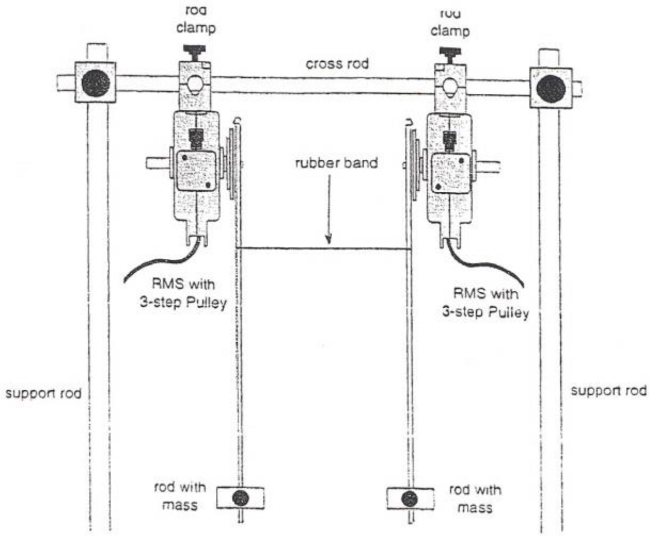
Objective:

To analyze and demonstrate the coupling of two pendulums hanging from a common stand first when attached by a rubber band, and then without. Also to observe the changes in coupling when phase differences were varied.

Description:



Two rotational Motion Sensors were attached to the cross rod which was the pendulum stand. The rotational Motion sensors were each then attached to a rod with a mass at the end. A single rubber band was wrapped around the two rods connecting them and effectively creating a spring. Both rotational Motion Sensors were attached to Data Studio and used to graph the motion of the rods. The masses were used to change the effective length of the pendulum by moving the center of mass up or down along the rod.

Theory:

From the equations of motion we have:

$$\frac{d^2}{dt^2}\theta_a + \frac{g}{l}\theta_a + \frac{K}{M}(\theta_a - \theta_b) = 0$$

And

$$\frac{d^2}{dt^2}\theta_b + \frac{g}{l}\theta_b + \frac{K}{M}(\theta_b - \theta_a) = 0$$

We then want to find values such that the pendulums follow the equations:

$$\theta_a = Acos(\omega t + \phi) \qquad \text{and} \qquad \theta_b = Bcos(\omega t + \phi)$$

Combining these equations gives:

$$\left(\frac{g}{l}+\frac{K}{M}-\omega^2\right)A-\frac{K}{M}B=0 \qquad \text{and} \qquad \left(\frac{g}{l}+\frac{K}{M}-\omega^2\right)B-\frac{K}{M}A=0$$

For the two-pendulum setup, the solutions for ω^2 are:

$$\omega_1^2=\frac{g}{l} \qquad \text{and} \qquad \omega_2^2=\frac{g}{l}+\frac{2K}{M}$$

Solving for these, we get the two normal modes for the coupled pendulums are (1, 1) and (1, -1) so $A_1 = B_1$ and $A_2 = -B_2$. To achieve equilibrium, the two modes are present in equal amounts in each pendulum. At this point all the amplitudes become equal and both pendula have equal energy i.e.

$$\theta_a = A\cos(\omega_1 t+\phi)+A\cos(\omega_2 t+\phi)$$

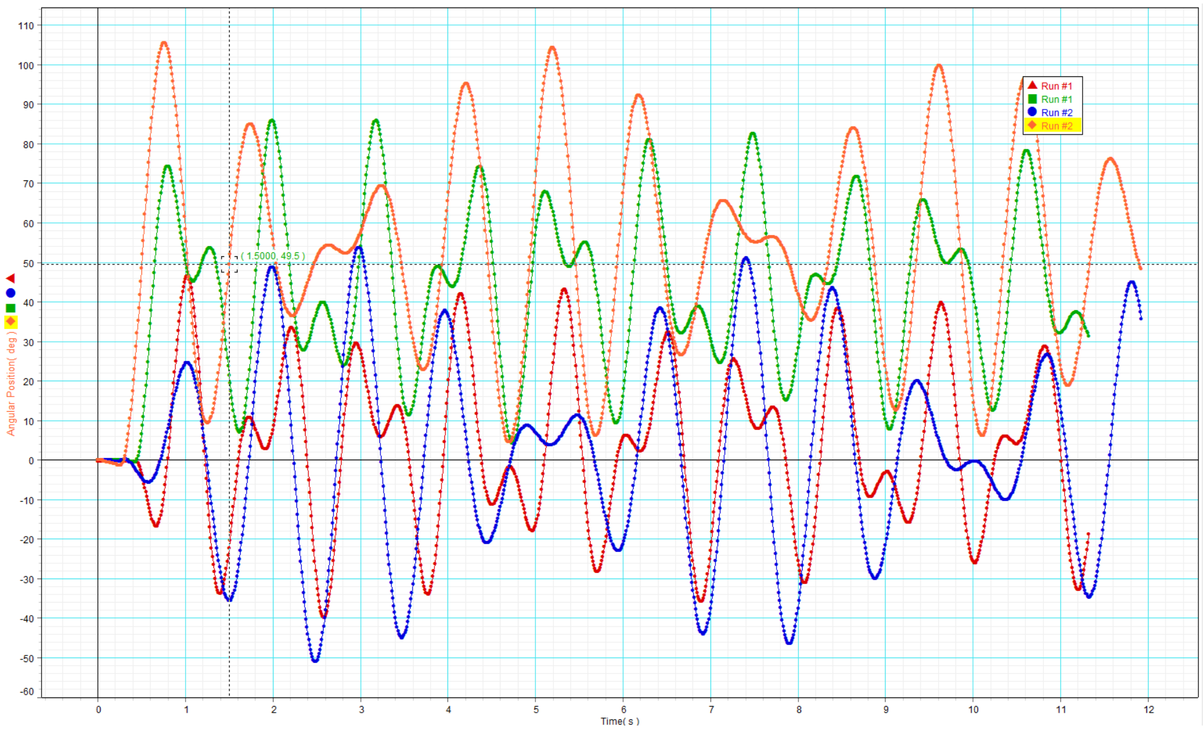
$$\theta_b = A\cos(\omega_1 t+\phi)-A\cos(\omega_2 t+\phi)$$

We can say that $\phi = 0$ so:

$$\theta_a = A\cos(\omega_1 t)+A\cos(\omega_2 t) \qquad \text{and} \qquad \theta_b = A\cos(\omega_1 t)+A\cos(\omega_2 t)$$

Which can then be written as:

$$\theta_a = 2A\cos(\omega_b t)\cos(\omega_{av} t) \qquad \text{and} \qquad \theta_b = 2A\sin(\omega_b t)\sin(\omega_{av} t)$$



Procedure and Data Analysis:

We started by matching the frequencies of the pendula as closely as possible using a sample rate of 10Hz to provide a suitable resolution of the oscillations. The frequency of the pendulums was found to be $(2.61 \pm 0.01)\text{radian. s}^{-1}$

The frequency given by the DataStudio sine-wave fit has units of radians per second.

The frequency of the symmetric mode is identical to the common frequency of the pendula when the rubber-band removed.

The frequency of the anti-symmetric mode is larger than the frequency of the symmetric mode. This is because the rubber-band pulls the pendula together when they reach their peak amplitudes i.e. the restoring force of the rubber-band reduces the time period or accelerates the pendulums (this was calculated to be 7.57 ± 0.01 radian.s^{-1}).

The value of K can be measured by hanging a mass off the rubber-band and measuring the displacement of the mass and using the equations of force.

The amplitude of one pendulum increases as that of the other decreases due to conservation of energy.

Moving the rubber-band further up or down along the pendulums changes the restoring force of the rubber-band acting on the pendulum.

The beat frequency was calculated to be 4.96 radian.s^{-1} using the two frequencies.

When the rubber-band is removed, the pendulums are still coupled but only very slightly. The remaining coupling is because of the common cross rod and the energy transfer through the rod.

Even with arbitrary initial conditions energy transfer takes place until the two pendula reach equilibrium.

For the original part of the experiment we examined the effect of moving rubber-band up and down on the rods.

	A1	A2	Height of the rubber-band from cross rod
1	52.1	56.4	9
2	45.1	43.5	14.5
3	34.1	35.0	18.3
4	35.2	34.5	24.5

Error Analysis:

We make a few assumptions in this experiment that make it inherently inaccurate for example:

- We assume the pendula are identical and have the same natural frequency. It is not experimentally viable to get the two pendulums to the same frequency and to be identical since inequalities in the locations of the center of mass, the differences in mass, differences in initial amplitude, different action of friction on the rotational motion sensors can all create variations in the frequency and any initial phase difference makes it hard to distinguish a difference in frequencies from a difference in phase.
- We assume the pendula have only one degree of freedom and oscillate in one plane only. The rubber-band used for most of the experiment adds an extra force that potentially creates an angle between the cross rod and he pendula i.e. the plane of oscillations could be at an angle that is not orthogonal to the cross rod. This changes the equations of force and energy transfer and creates uncertainties in the experimental results.
- We assume the pendulum rods are massless so the masses of the pendulum rods remain unaccounted for. This assumption allows us to assume that hanging the

masses from an equivalent height from the pivot should give us equal effective lengths and therefore equal frequencies, but the rods (which do have mass) could in fact have different masses and manufacture (or wear and tear) defects could create discrepancies in this assumption too.

- We assumed that the angles are small and the equations of motion can be linearized using $sin\theta \cong \theta$. This assumption is inaccurate since the angles were varied by up to 60 degrees where this assumption no longer holds. This makes the equations we used incomplete in this experiment.
- We assume there is no friction or loss of energy in the system. The loss of energy becomes apparent very soon on the graphs in the damping of the pendula. This too is not accounted for in the experiment.